

Σ の公式

公式 (1) $\sum_{k=1}^n c = nc$ (c は定数)

公式 (2) $\sum_{k=1}^n k = \frac{1}{2}n(n+1)$

公式 (3) $\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$

公式 (4) $\sum_{k=1}^n k^3 = \left(\frac{1}{2}n(n+1)\right)^2$

計算例

公式 (1) $\sum_{k=1}^n c = nc$ (c は定数)

$$\sum_{k=1}^n 4 = \underbrace{4 + 4 + 4 + \cdots + 4}_{n \text{ 個}} = 4n \quad \boxed{\text{答}}$$

$$\sum_{k=1}^7 3 = \underbrace{3 + 3 + 3 + 3 + 3 + 3 + 3}_{7 \text{ 個}} = 21 \quad \boxed{\text{答}}$$

計算例

$$\text{公式 (2)} \quad \sum_{k=1}^n k = \frac{1}{2}n(n+1)$$

$n \rightarrow 8$ と置き換える

$$\begin{aligned} \sum_{k=1}^8 k &= \frac{1}{2} \cdot 8 \cdot (8 + 1) \\ &= \frac{1}{2} \cdot 8 \cdot 9 = 36 \quad \boxed{\text{答}} \end{aligned}$$

計算例

$$\text{公式 (2)} \quad \sum_{k=1}^n k = \frac{1}{2}n(n+1)$$

$n \rightarrow n+1$ と置き換える

$$\begin{aligned} \sum_{k=1}^{n+1} k &= \frac{1}{2}(n+1)\left((n+1)+1\right) \\ &= \frac{1}{2}(n+1)(n+2) \quad \boxed{\text{答}} \end{aligned}$$

計算例

$$\text{公式 (3)} \quad \sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$$

$n \rightarrow 7$ と置き換える

$$\begin{aligned} \sum_{k=1}^7 k^2 &= \frac{1}{6} \cdot 7 \cdot (7+1) \cdot (2 \cdot 7 + 1) \\ &= \frac{1}{6} \cdot 7 \cdot 8 \cdot 15 = 140 \quad \boxed{\text{答}} \end{aligned}$$

計算例

$$\text{公式 (4)} \quad \sum_{k=1}^n k^3 = \left(\frac{1}{2} n(n+1) \right)^2$$

$n \rightarrow 10$ と置き換える

$$\begin{aligned} \sum_{k=1}^{10} k^3 &= \left(\frac{1}{2} \cdot 10 \cdot (10 + 1) \right)^2 \\ &= \left(\frac{1}{2} \cdot 10 \cdot 11 \right)^2 = 55^2 = 3025 \quad \boxed{\text{答}} \end{aligned}$$