

Σ の性質

$$\text{性質 (1)} \quad \sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

$$\sum_{k=1}^n (k^2 + k - 5) = \sum_{k=1}^n k^2 + \sum_{k=1}^n k - \sum_{k=1}^n 5$$

【分けてよい】

Σ の性質

性質 (2) $\sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k$ (c は定数)

$$\sum_{k=1}^n 8k = 8 \sum_{k=1}^n k = 8 \times \sum_{k=1}^n k \quad \text{【のこと】} = 8 \cdot \sum_{k=1}^n k$$

【数字は前に出してよい】

計算例

公式 (1) $\sum_{k=1}^n c = nc$ (c は定数)

公式 (2) $\sum_{k=1}^n k = \frac{1}{2}n(n+1)$

$$\begin{aligned}\sum_{k=1}^n (4k + 3) &= \sum_{k=1}^n 4k + \sum_{k=1}^n 3 && \leftarrow \text{【分けてよい】} \\ &= 4 \sum_{k=1}^n k + 3n && \leftarrow \text{【数字は前に出してよい】} \\ &= 4 \cdot \frac{1}{2}n(n+1) + 3n \\ &= 2n(n+1) + 3n \\ &= n(2(n+1) + 3) && \leftarrow \text{【}n\text{ でくくる】}\end{aligned}$$

計算例

$$= n(2(n+1) + 3)$$

$$= n(2n + 2 + 3)$$

$$= n(2n + 5) \quad \boxed{\text{答}}$$

計算例

$$\sum_{k=1}^n c = nc$$

$$\sum_{k=1}^n k = \frac{1}{2}n(n+1)$$

$$\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{k=1}^n (k-2)(k-3) = \sum_{k=1}^n (k^2 - 5k + 6) \quad \leftarrow \text{【展開する】}$$

【分けてよい/数字は前】 \rightarrow

$$= \sum_{k=1}^n k^2 - 5 \sum_{k=1}^n k + \sum_{k=1}^n 6$$

$$= \frac{1}{6}n(n+1)(2n+1)$$

$$- 5 \cdot \frac{1}{2}n(n+1) + 6n$$

計算例

$$\begin{aligned} &= \frac{1}{6}n(n+1)(2n+1) - \frac{5}{2}n(n+1) + 6n \\ &= \frac{1}{6}n\left((n+1)(2n+1) - 15(n+1) + 36\right) \\ &= \frac{1}{6}n\left(2n^2 + 3n + 1 - 15n - 15 + 36\right) \\ &= \frac{1}{6}n(2n^2 - 12n + 22) \end{aligned}$$

計算例

$$= \frac{1}{6}n(2n^2 - 12n + 22)$$

$$= \frac{1}{6}n \cdot 2(n^2 - 6n + 11)$$

$$= \frac{1}{3}n(n^2 - 6n + 11) \quad \boxed{\text{答}}$$

Σ のかけ算はダメです

$$\begin{aligned} & \sum_{k=1}^n (k-2)(k-3) \\ = & \sum_{k=1}^n (k-2) \times \sum_{k=1}^n (k-3) \quad \text{はダメ！} \end{aligned}$$

計算例

$$\sum_{k=1}^n k = \frac{1}{2}n(n+1)$$

$$\sum_{k=1}^n k^3 = \left(\frac{1}{2}n(n+1)\right)^2$$

$$\begin{aligned}\sum_{k=1}^n (k^3 - k) &= \sum_{k=1}^n k^3 - \sum_{k=1}^n k \\ &= \left(\frac{1}{2}n(n+1)\right)^2 - \frac{1}{2}n(n+1) \\ &= \frac{1}{2}n(n+1) \left(\frac{1}{2}n(n+1) - 1\right) \\ &= \frac{1}{2}n(n+1) \cdot \frac{1}{2} \left(n(n+1) - 2\right)\end{aligned}$$

計算例

$$= \frac{1}{2}n(n+1) \cdot \frac{1}{2}(n(n+1) - 2)$$

$$= \frac{1}{2}n(n+1) \cdot \frac{1}{2}(n^2 + n - 2)$$

$$= \frac{1}{4}n(n+1)(n^2 + n - 2)$$

$$= \frac{1}{4}n(n+1)(n-1)(n+2) \quad \boxed{\text{答}}$$